Dissipation and noise in EFTs

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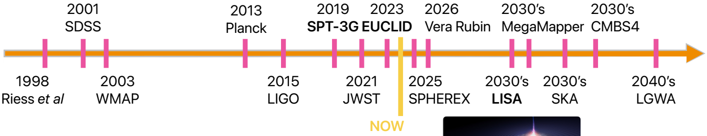
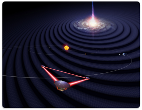
Aspirations

Learn about **fundamental physics**:

- New degrees of freedom;
- GR at high energy;
- QFT in curved spacetimes;
- Quantum gravity; $\cdots$

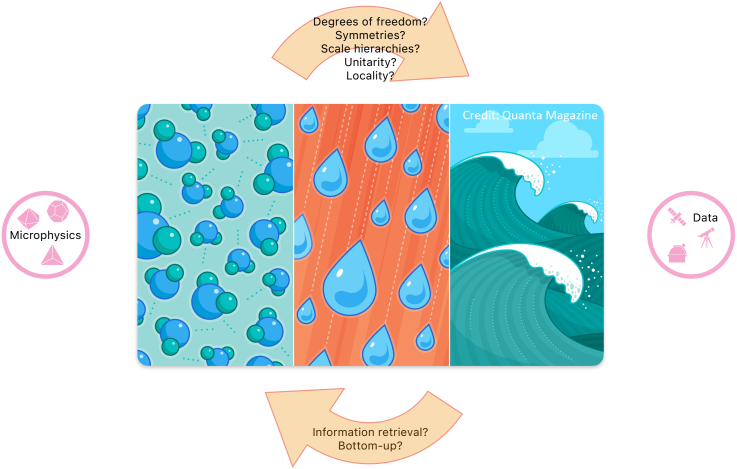
A defining moment for **cosmology**:



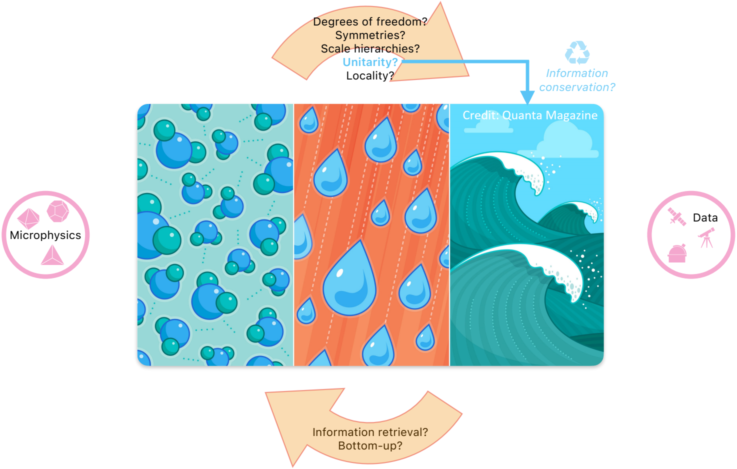


Need to organise dialogue between theory and observations

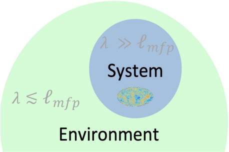
Effective Field Theories



Effective Field Theories



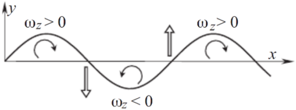
When (non)-unitarity matters



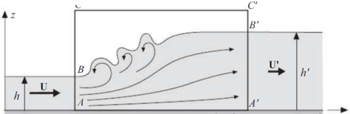
System interacts with environment:
$\Rightarrow$ non-unitary evolution

Dissipation & noise $=$ energy & information losses

Perfect fluid: Wilsonian EFT



Imperfect fluid: non-equilibrium EFT



Credit: [Guyon *et al*, 2012]

What about cosmology?

- **Observable** universe $\neq$ whole; 
- Continuously **evolving**; 
- Always a **medium**; 
- $\exists$ **fluxes** between sectors. 

$\Rightarrow$ effective dynamics often non-unitary [T.C., J. Grain, V. Vennin, 2212.09486]

Goal: EFTs accounting for dissipation & noise in cosmology

Combining EFTs and Open Quantum Systems

Langevin equation

$$\mathrm{d}\mathcal{O} = \{H, O\}\mathrm{d}t + \mathrm{d}\xi$$

Martin-Siggia-Rose formalism

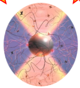
$$P = \int \mathcal{D}x\mathcal{D}\widetilde{x}e^{-S[x,\widetilde{x}]}$$

Fokker-Planck equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mathcal{L}_{\mathrm{FP}}[P]$$

Stochastic Schrödinger equation

$$\mathrm{d}\widehat{\mathcal{O}} = -i\left[\widehat{H}, \widehat{\mathcal{O}}\right] + \mathrm{d}\widehat{\xi}$$



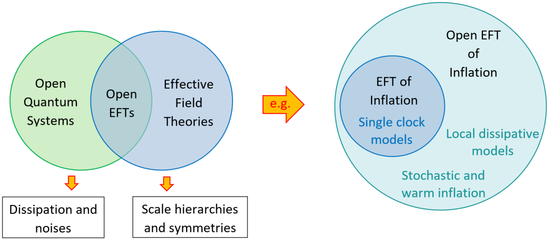
Influence functional

$$\langle\phi_1|\,\widehat{\rho}_{\mathrm{red}}\,|\phi_2\rangle = \int \mathcal{D}\phi_\pm e^{iS_{\mathrm{IF}}[\phi_+,\phi_-]}$$

Master equation

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{L}[\widehat{\rho}_{\mathrm{red}}]$$

Extend the **embedding power** of EFTs:

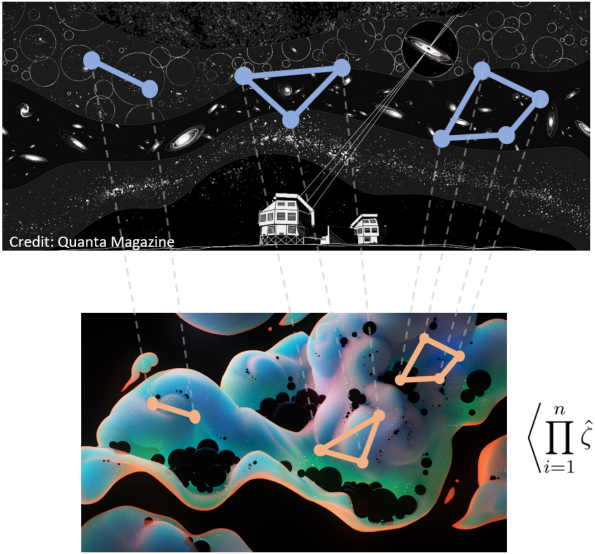


Outline

1. Open inflation
2. Open E&M
3. Open gravity

Outline

Cosmological correlators



$$\left\langle \prod_{i=1}^{n} \delta(\boldsymbol{k}_i) \right\rangle$$

$$\left\langle \prod_{i=1}^{n} \hat{\zeta}(\boldsymbol{k}_i, \eta_0) \right\rangle$$

Schwinger-Keldysh formalism

Consider some **observable**

$$\widehat{Q} \equiv \widehat{\zeta}(\boldsymbol{x}_1)\widehat{\zeta}(\boldsymbol{x}_2)\cdots\widehat{\zeta}(\boldsymbol{x}_n)$$

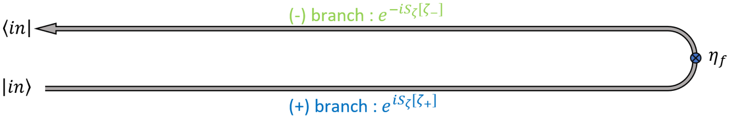
and some unitary **evolution operator** $\widehat{U}(\eta,\eta_0)$ so that

$$|\Psi(\eta)\rangle = \widehat{U}(\eta,\eta_0)\,|\mathrm{BD}\rangle \quad \text{with} \quad \langle\zeta|\,\widehat{U}(\eta,\eta_0)\,|\zeta_1\rangle = \int_{\zeta_1}^{\zeta}\mathcal{D}\left[\Phi\right]e^{iS[\Phi]}.$$

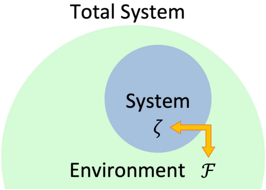
If $S\left[\Phi\right] = S_\zeta\left[\zeta\right]$, see [Donath & Pajer, 2402.05999]:

$$\left\langle\widehat{Q}(\eta)\right\rangle = \int \mathrm{d}\zeta\mathrm{d}\zeta_1\mathrm{d}\zeta_2\left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right]\left[\langle\zeta|\,\widehat{U}(\eta,\eta_0)\,|\zeta_1\rangle\right]\left[\langle\zeta_1|\mathrm{BD}\rangle\langle\mathrm{BD}|\zeta_2\rangle\right]\left[\langle\zeta_2|\,\widehat{U}^\dagger(\eta,\eta_0)\,|\zeta\rangle\right]$$

$$= \int \mathrm{d}\zeta\mathrm{d}\zeta_1\mathrm{d}\zeta_2\left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right]\int_{\zeta_1}^{\zeta}\mathcal{D}\left[\zeta_+\right]\int_{\zeta_2}^{\zeta}\mathcal{D}[\zeta_-]e^{iS_\zeta[\zeta_+]-iS_\zeta[\zeta_-]}\langle\zeta_1|\mathrm{BD}\rangle\langle\mathrm{BD}|\zeta_2\rangle$$



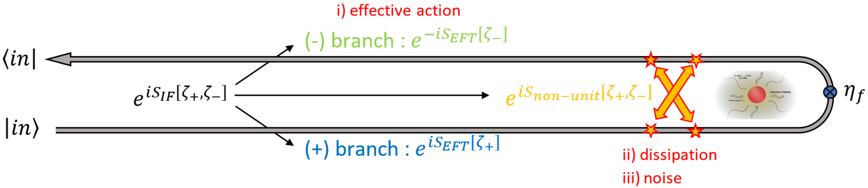
Integrating out an environment



- $S[\Phi] = S_\zeta[\zeta] + S_{\mathcal{F}}[\mathcal{F}] + S_{\mathrm{int}}[\zeta;\mathcal{F}]$ with $\mathcal{F}$ a **hidden sector**.
- *Goal:* tracing out $\mathcal{F}$, the environment being **unobservable**.

Effects of the environment captured by the **Influence Functional** (IF):

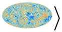
$$\left\langle \widehat{Q}(\eta) \right\rangle = \int \mathrm{d}\zeta \mathrm{d}\zeta_1 \mathrm{d}\zeta_2 \left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right] \int_{\zeta_1}^{\zeta} \mathcal{D}[\zeta_+] \int_{\zeta_2}^{\zeta} \mathcal{D}[\zeta_-] e^{iS_\zeta[\zeta_+]-iS_\zeta[\zeta_-]+\boldsymbol{iS_{\mathrm{IF}}[\zeta_+;\zeta_-]}}$$



What are the rules obeyed by $S_{\mathrm{IF}}[\zeta_+;\zeta_-]$*?*

The Open EFT of Inflation [S.A. Agüí Salcedo, T.C. & E. Pajer, 2404.15416]

Early universe: one scalar degree of freedom $\pi(\boldsymbol{x}, t)$:

Observed ⟨  ⟩ ⇔ $\langle \hat{\pi}^n \rangle(t) = \int \mathrm{d}\pi \pi^n \mathrm{Prob}_\pi(t)\,.$

- EFT of Inflation [Cheung *et al.*, 2008]: most generic wavefunction

$$\mathrm{Prob}_\pi(t) = |\Psi_\pi(t)|^2 = \left| \int_\Omega^\pi \mathcal{D}\pi \, e^{iS_{\mathrm{eff}}[\pi]} \right|^2$$

- **Dissipation** & **noise**: most generic density matrix

$$\mathrm{Prob}_\pi(t) = \rho_{\pi\pi}(t) = \int_\Omega^\pi \mathcal{D}\pi_+ \int_\Omega^\pi \mathcal{D}\pi_- \, e^{iS_{\mathrm{eff}}[\pi_+, \pi_-]}$$

Physical principles restrict $S_{\mathrm{eff}}[\pi_+, \pi_-]$:

1. Unitarity: {Sys. + Env.} closed ⇒ non-equilibrium constraints;
2. Symmetries: in-in coset construction;
3. Locality: truncatable power counting scheme.

Non-equilibrium constraints [Liu & Glorioso, 2018]

Requiring **Open QFT** originates from a unitary "closed" UV theory:

$$\text{i)}\ \mathrm{Tr}[\widehat{\rho}] = 1, \qquad \text{ii)}\ \widehat{\rho}^{\dagger} = \widehat{\rho} \qquad \text{and} \qquad \text{iii)}\ \widehat{\rho} \geq 0$$

implies constraints on $S_{\rm eff}\left[\pi_+, \pi_-\right] \equiv S_{\rm unit}[\pi_+] - S_{\rm unit}[\pi_-] + S_{\rm non-unit}[\pi_+, \pi_-]$:

$$\begin{aligned}
&\text{i)} && S_{\rm eff}\left[\pi_+, \pi_+\right] = 0, && S_{\rm eff}\left[\pi_r, \pi_a = 0\right] = 0; \\
&\text{ii)} && S_{\rm eff}\left[\pi_+, \pi_-\right] = -S^*_{\rm eff}\left[\pi_-, \pi_+\right], && S_{\rm eff}\left[\pi_r, \pi_a\right] = -S^*_{\rm eff}\left[\pi_r, -\pi_a\right]; \\
&\text{iii)} && \mathfrak{Im} S_{\rm eff}\left[\pi_+, \pi_-\right] \geq 0, && \mathfrak{Im} S_{\rm eff}\left[\pi_r, \pi_a\right] \geq 0,
\end{aligned}$$

for $\pi_r = (\pi_+ + \pi_-)/2$ and $\pi_a = \pi_+ - \pi_-$. *Consequences*:

1. $S_{\rm eff}\left[\pi_r, \pi_a\right]$ starts **linear** in π_a;
2. **Odd** powers of π_a are purely real; **even** powers of π_a purely imaginary;
3. Positivity bounds on the **noise coefficients**.

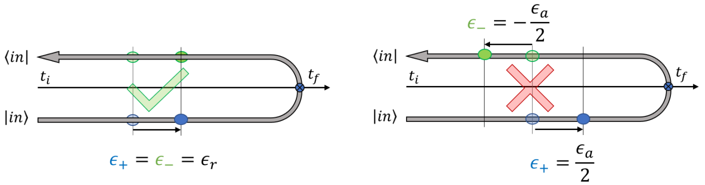
$\Rightarrow$ *Already reduce the scope of available Open EFTs*

In-in coset construction [Hongo *et al.*, 2018], [Akyuz, Goon & Penco, 2023]

Two **simplifications** from [Cheung *et al.*, 2008]: [regime of validity? ⇒ later]

1. *Decoupling limit*: Mixing $\pi/\delta g$ small as long as $E \sim H \gg E_{\rm mix} \sim \epsilon^{1/2} H$

⇒ **enough** to construct theory of **dissipative shift symmetric scalar**:



$S_{\rm eff}[\pi_r, \pi_a]$ invariant under *retarded time diffeomorphism*:

$$t \to t + \epsilon_r : \qquad \pi_r \to \pi_r - \epsilon_r, \qquad \pi_a \to \pi_a.$$

<u>*Building blocks:*</u> π_a, $t + \pi_r$, $\partial_\mu \pi_a$, $\partial_\mu(t + \pi_r)$.

2. *Derivative expansion*: locality and truncatable power counting scheme.

Effective functional

Decoupling limit + derivative expansion (up to one ∂/field):

- *Quadratic order:* $1 \to 5$ EFT param (1 tadpole constraint):

$$S_{\text{eff}}^{(2)} = \int \mathrm{d}^4x\sqrt{-g}\Big\{\overbrace{\dot\pi_r\dot\pi_a - c_s^2\partial_i\pi_r\partial^i\pi_a}^{\text{Kinetic term}} \underbrace{-\gamma\dot\pi_r\pi_a}_{\text{Dissipation}} + i\underbrace{\left[\beta_1\pi_a^2 - \left(\beta_2 - \beta_4\right)\dot\pi_a^2 + \beta_2\left(\partial_i\pi_a\right)^2\right]}_{\text{Noise}}\Big\}$$

- *Cubic order:* $1 \to 13$ EFT param: EFToI famous for relating operators at different orders because of **non-linearly realised boosts** [López Nacir *et al.*, 2011].

$$\text{EFToI}: \quad \mathcal{L} \supset \left(c_s^2 - 1\right)\left[-2\dot\pi_r + \left(\partial_\mu\pi_r\right)^2\right]\dot\pi_a$$

$$\text{Dissipation}: \quad \mathcal{L} \supset \gamma\left[-2\dot\pi_r + \left(\partial_\mu\pi_r\right)^2\right]\pi_a$$

$$\text{Noise}: \quad \mathcal{L} \supset i\beta_4\left(-\dot\pi_a + \partial_\mu\pi_r\partial^\mu\pi_a\right)^2$$

Recover and extend EFToI construction.

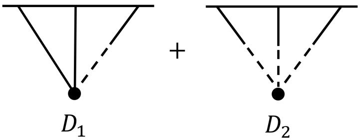
Standard observables

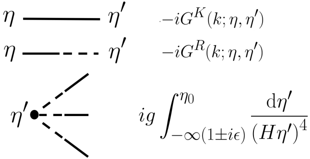
Symmetries ensure existence of nearly scale invariant power spectrum

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle = \frac{H^2}{f_\pi^4} \langle \pi^c_{\boldsymbol{k}} \pi^c_{\boldsymbol{k}'} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k} + \boldsymbol{k}) \frac{2\pi^2}{k^3} \Delta^2_\zeta(k).$$

$\Rightarrow \Delta^2_\zeta = 10^{-9}$ obtained by imposing hierarchies of scales.

Bispectrum computed in **perturbation theory** using standard in-in rules.

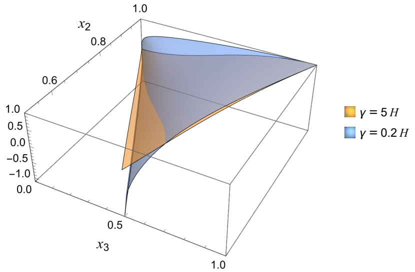




$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = -\frac{H^3}{f_\pi^6} \langle \pi^c_{\boldsymbol{k}_1} \pi^c_{\boldsymbol{k}_2} \pi^c_{\boldsymbol{k}_3} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B(k_1, k_2, k_3).$$

$$S(x_2, x_3) \equiv (x_2 x_3)^2 \frac{B(k_1, x_2 k_1, x_3 k_1)}{B(k_1, k_1, k_1)}, \qquad f_{\mathrm{NL}}(k_1, k_2, k_3) \equiv \frac{5}{6} \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + 2 \text{ perms.}}.$$

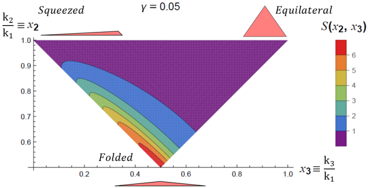
Bispectrum shapes

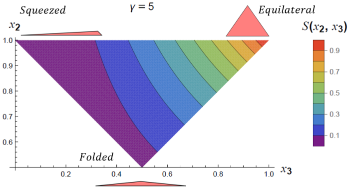


Main features:

- $\gamma \gg H$: equilateral;
- $\gamma \ll H$: folded;
- Consistency relations;
- Regularized divergence.

Consistent with **flat-space/sub-Hubble** analytic results:



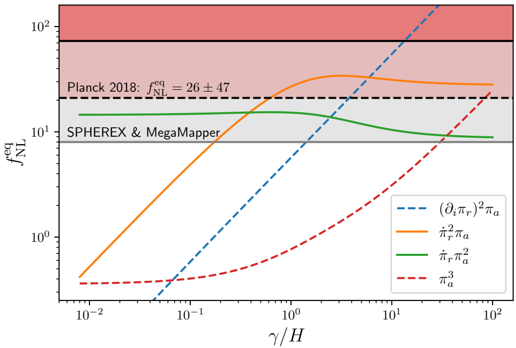


Matching and $f_{\rm NL}$ with [Creminelli *et al.*, 2305.07695]

UV completion: inflaton ϕ + massive scalar field χ with softly-broken $U(1)$:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}\left(\partial\phi\right)^2 - V(\phi) - \left|\partial\chi\right|^2 + M^2\left|\chi\right|^2 - \frac{\partial_\mu\phi}{f}\left(\chi\partial^\mu\chi^* - \chi^*\partial^\mu\chi\right) - \frac{1}{2}m^2\left(\chi^2 + \chi^{*2}\right)\right].$$

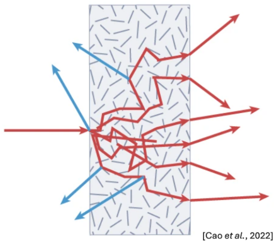
$\Rightarrow$ narrow **instability band** in sub-Hubble regime: *local* particle production.



Outline

Open Electromagnetism [S.A. Agüí Salcedo, T.C. & E. Pajer, 2412.12299]

Dissipative theory for a **massless spin 1 photon**: theory of light in a medium.

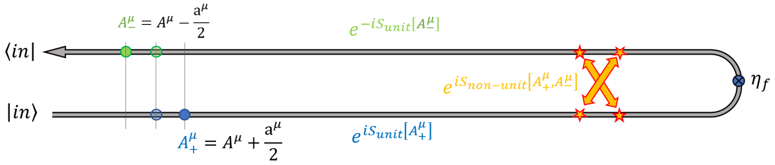


- *dielectric material* (insulator):
  - 2 transverse d.o.f.
  - gauge invariance:

$$A^\mu_\pm \to A^\mu_\pm + \partial^\mu \epsilon_\pm$$

- *relax IR unitarity* = includes dissipation & noise.

Keldysh basis: retarded $A^\mu = \left(A^\mu_+ + A^\mu_-\right)/2$; advanced $a^\mu = A^\mu_+ - A^\mu_-$.



Retarded & advanced gauge transformation

Retarded gauge transformation $\epsilon_+ = \epsilon_- = \epsilon_r$:

$$A^\mu \to A^\mu + \partial^\mu \epsilon_r \,, \qquad\qquad a^\mu \to a^\mu \,.$$

Advanced gauge transformation $\epsilon_+ = -\epsilon_- = \epsilon_a$:

$$A^\mu \to A^\mu \,, \qquad\qquad a^\mu \to a^\mu + \partial^\mu \epsilon_a \,.$$

Principles: i) NEQ constraints, ii) locality and iii) retarded gauge invariance.

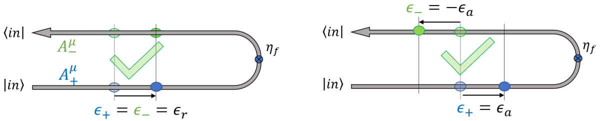
Effective functional constructed out of $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and a^μ:

$$S_1 = \int_{\omega,\boldsymbol{k}} \left[a^0 i k_i F^{0i} + a_i \left(\gamma_2 F^{0i} + \gamma_3 i k_j F^{ij} + \gamma_4 \epsilon^i{}_{jl} F^{jl} \right) + a^\mu j_\mu \right]$$

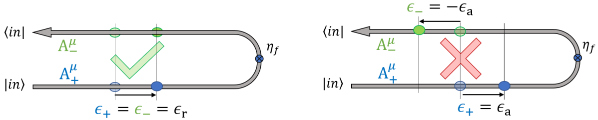
$$S_2 = \int_{\omega,\boldsymbol{k}} i a^\mu N_{\mu\nu} a^\nu \,, \qquad S_{n\geq 3} = \cdots$$

Summary

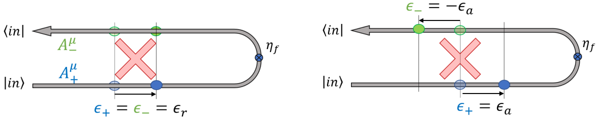
1. **Unitary**: $\Delta S_{\text{eff}}^{\text{adv}} = 0 \quad \Rightarrow \quad \partial^\mu j_\mu = 0$: recover Maxwell in medium;



2. **Non-unitary**: $\Delta S_{\text{eff}}^{\text{adv}} \neq 0 \quad \Rightarrow \quad \partial^\mu j_\mu \neq 0$: noise constraint;



3. **Conductor**: $\Delta S_{\text{eff}}^{\text{ret}} \neq 0 \quad \Rightarrow$ new d.o.f.: Proca mass term.



Dispersion relations

Gauge fixing

- retarded Coulomb gauge: $\partial_i A^i = 0$

$$\exists\ \epsilon_r \quad \text{s.t.} \quad k_i A'^i = 0, \quad \text{where} \quad A'^\mu = A^\mu + \epsilon_r k^\mu .$$

- advanced Coulomb gauge: $\partial_i a^i = 0$

$$\exists\ \epsilon_a \quad \text{s.t.} \quad k_i a'^i = 0, \quad \text{where} \quad a'^\mu = a^\mu + \epsilon_a v^\mu .$$

Eigenvalues of the kinetic matrix: 1 constrained dof, 2 propagating dof

$$\left(k^2\, , i\gamma_2\omega + \gamma_3 k^2 + 2\gamma_4 k, i\gamma_2\omega + \gamma_3 k^2 - 2\gamma_4 k\right) .$$

Introduce $\gamma_2 = \Gamma - i\omega$, $\gamma_3 = -c_s^2$:

$$\omega^2 + i\Gamma\omega - c_s^2 k^2 \pm 2\gamma_4 k = 0 \quad \Rightarrow \quad \omega = -i\frac{\Gamma}{2} \pm \sqrt{c_s^2 k^2 - (\Gamma/2)^2 \mp 2\gamma_4 k} .$$

Recovering electromagnetism in a medium

From S_{eff}, obtain modified Gauss and Ampère laws:

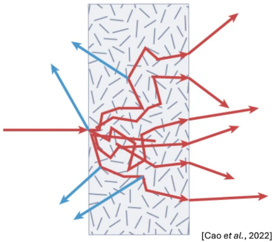
$$\frac{\delta S_{\text{eff}}}{\delta a^0} = 0 \qquad \Rightarrow \qquad \nabla.\mathbf{E} = j_0 + \xi_0,$$
$$\frac{\delta S_{\text{eff}}}{\delta a^i} = 0 \qquad \Rightarrow \qquad \gamma_2 \mathbf{E} + \gamma_3 \nabla \times \mathbf{B} - 2\gamma_4 \mathbf{B} = \mathbf{j} + \xi,$$

and a noise constraint: *charge non-conservation in the system*

$$ik^\mu (j_\mu + \xi_\mu) = (i\omega + \gamma_2)(j_0 + \xi_0).$$

Properties:

- Dispersive medium: $n = 1/\sqrt{-\gamma_3}$;
- Dissipative medium: $\gamma_2 = -i\omega + \Gamma$;
- Anisotropic medium: γ_4;
- Random medium: ξ^μ.

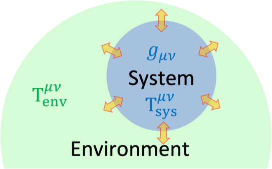


[Cao *et al.*, 2022]

Outline

Open gravity with S. Agui-Salcedo, L. Duffner, F. McCarthy & E. Pajer *[to appear]*

Dissipative theory for a **massless spin** 2 **graviton**: theory of gravity in a medium.



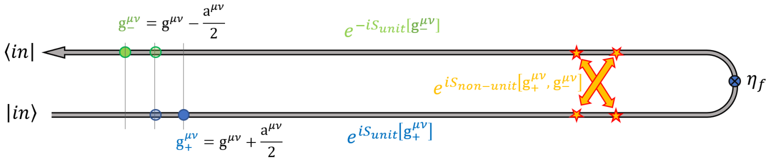
Diffeomorphisms invariance:

$$g_{\pm}^{\mu\nu}(x) \to \frac{\partial(x^\mu + \xi^\mu_\pm)}{\partial x^\alpha}\frac{\partial(x^\nu + \xi^\nu_\pm)}{\partial x^\beta} g_\pm^{\alpha\beta}(x)$$

for each branch of SK path integral contour.

Keldysh basis for **metric perturbations**:

$$g = \frac{g_+ + g_-}{2} = \bar{g} + \delta g, \qquad \text{and} \qquad a = g_+ - g_- = \delta a$$



Single-clock cosmology

$S_{\rm eff}[g_{\mu\nu}, a^{\mu\nu}]$ invariant under retarded spatial diffeomorphisms:

$$S_{\rm eff}[g_{\mu\nu}, a^{\mu\nu}] = \int \mathrm{d}^4x \sqrt{-g} \left[M_{\mu\nu} a^{\mu\nu} + i N_{\mu\nu\rho\sigma} a^{\mu\nu} a^{\rho\sigma} + \cdots \right]$$

with $M_{\mu\nu}$ and $N_{\mu\nu\rho\sigma}$ rank-2 and 4 cotensors. Up to 2nd order in derivatives:

$$M_{00} = \gamma_1^{tt} R + \gamma_2^{tt} R^{00} + \gamma_3^{tt} K + \gamma_4^{tt} K^2 + \gamma_5^{tt} K_{\alpha\beta} K^{\alpha\beta} + \gamma_6^{tt} \nabla^0 K + \gamma_7^{tt} \,;$$

$$M_{0i} = \gamma_1^{ts} R^0{}_i + \gamma_2^{ts} \nabla_i K + \gamma_3^{ts} \nabla_\beta K^\beta{}_i \,;$$

$$M_{ij} = g_{ij} \left(\gamma_1^{ss} R + \gamma_2^{ss} R^{00} + \gamma_3^{ss} K + \gamma_4^{ss} K^2 + \gamma_5^{ss} K_{\alpha\beta} K^{\alpha\beta} + \gamma_6^{ss} \nabla^0 K + \gamma_7^{ss} g \right)$$
$$+ \gamma_8^{ss} R_{ij} + \gamma_9^{ss} R_i{}^0{}_j{}^0 + \gamma_{10}^{ss} K_{ij} + \gamma_{11}^{ss} \nabla^0 K_{ij} + \gamma_{12}^{ss} K_{i\alpha} K^\alpha{}_j + \gamma_{13}^{ss} K K_{ij} + \gamma_{14}^{ss} g_{ij} \,.$$

and similarly for $N_{\mu\nu\rho\sigma}$.

Retarded and *advanced* Stueckelberg tricks ⇒ systematic construction.

A glimpse on what to expect

- Dissipative and stochastic Einstein Equations: $G_{\mu\nu} + \Gamma \mathcal{D}_{\mu\nu} = T_{\mu\nu} + \xi_{\mu\nu}$
- Non-conserved stress-energy tensor: $\nabla_\mu T^{\mu\nu} \neq 0$

Phenomenology: [data analysis ⇒ F. McCarthy (ACT/SO)]

- **Background**: Interacting DE/DM sectors

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = \Gamma \quad \text{and} \quad \dot{\rho}_m + 3H(\rho_m + p_m) = -\Gamma$$

- **Clustering** ⇒ redshift space distortion (RSD) and weak lensing (WL)

$$k^2 \langle \psi \rangle = -4\pi G \mu(a,k) a^2 \rho_m \langle \delta \rangle, \qquad k^2 \frac{\langle \psi + \phi \rangle}{2} = -4\pi G \Sigma(a,k) a^2 \rho_m \langle \delta \rangle$$

- **Gravitational waves** ⇒ GW production, propagation and dissipation

$$\ddot{h}_{ij} + \Gamma \dot{h}_{ij} + c_t^2 h_{ij} + \theta \epsilon_{ilm} k_m h_{jl} = T_{ij} + \xi_{ij}$$

Rich phenomenology to explore,
*eventually **already constrained from data**.*

Conclusion

Open inflation:

1. Systematic EFT accounting for local **dissipation** and **noise**;
2. Smoking gun near **folded triangles** in **primordial non-Gaussianities**.

Beyond decoupling, tensor modes, entropy bounds, $\cdots$

Open E&M:

1. Sandbox for Open EFTs with **gauge symmetries**;
2. Dissipation and noise constrained by symmetries, even **out-of-equilibrium**.

Conductors, non-linear, non-Abelian, anomaly, $\cdots$

Open gravity:

1. **Diffeomorphism invariance** constrain dissipation and noise;
2. Background evolution and cosmological inhomegeneities tight together.

Dark energy, gravitational waves, galaxy clustering, $\cdots$